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by

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Status Report to the

Lehigh Project Subcommittee

Of the Welding Research Council

PLASTIC MOMENT BALANCING FOR UNBRACED MULTI-STORY FRAMES

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SYNOPSIS

This report gives a brief historical survey of the plastic moment balancing method. An evaluation of three plastically designed unbraced frames from the sales engineering and the structural engineering viewpoint is also included. This evaluation serves to indicate the need for a flexible and adaptable design method for unbraced multi-story frames. It is tentatively concluded that the plastic moment balancing method satisfies this need.

I. PLASTIC MOMENT BALANCING (BRIEF HISTORICAL SURVEY)

A. 1954 - Plastic Moment Distribution (Ref. 1 and 2)

Discussion of Ref. 1 indicated that a variation of this method was used in France "40 years ago."

BT

B. 1961 - Plastic Moment Balancing (Ref. 3)

Used example to demonstrate analysis of unbraced multi-story frame under combined loading. Steps included

1. Girder equilibrium
2. Joint equilibrium
3. Story equilibrium - omitted $P\Delta$ effect

C. 1961 - Design of Gabled and Multi-Story Frames by Plastic Moment Distribution (Ref. 4)

Obtained bounds on plastic moment capacity for frame members using weak-beam-strong-column and strong-beam-weak-column plastic moment distributions. Sway effects neglected. Most column sizes limited by Formula (20) in Part 2 of the AISC Specification. This conservative formula intends to control elastic-plastic sidesway buckling under gravity loading.

D. 1963 - Design Methods Memo (Ref. 5)

Indicated how to include $P\Delta$ effect in story equilibrium condition.

E. 1964 - Minimum Weight Plastic Design of Continuous Frames (Ref. 6)

Used plastic mechanism and plastic moment distribution (Ref. 1) for minimum weight analysis. Included multi-story frame design example but did not consider $P\Delta$ effect. This was considered in discussion (Ref. 7). Author's closure (Ref. 8) indicated that weight not sensitive to different distributions of plastic moment capacity close to theoretical minimum weight distribution.

F. 1965 - Summer Conference on Plastic Design of Multi-Story Frames (Ref. 9)

Notes included a basic description of plastic moment balancing for unbraced multi-story frames together with design examples and sway deflection approximations. This method appropriate for manual calculations. Girders designed for clear span.

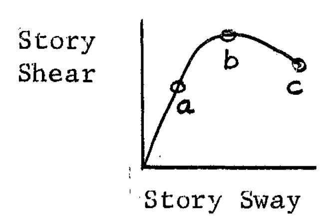
G. 1966 - Optimum Design of Multi-Story Frames by Plastic Theory (Ref. 10)

Used plastic moment balancing to minimize a plastic moment weight function for a 3-bay frame by iteration on a digital computer. Hypothetical increase in story shear used to account for PΔ effect. Provisions for enforcing girder depth limitations included.

H. 1966 - Preliminary Design of Unbraced Multi-Story Frames (Ref. 11)

Explains plastic moment balancing including the following developments:

1. PΔ effects included using estimated sway at
   - a. Working load, or
   - b. Ultimate load, or
   - c. Mechanism load



2. Girders designed for clear span.
3. Columns designed for clear height.
4. Vertical distribution factors for sway moments in a story.

5. Horizontal distribution factors for sway moments on a level.
6. Restricted hinge patterns to control sway effects.
7. Statical consideration of composite girders and girders reinforced with haunches or stubs for wind.
8. Computer program for preliminary design. Designer may specify "standard" parameters for items 2 to 7 or may vary input to consider refined options. Two stage FORTRAN program, suitable for a small to intermediate size computer (for example GE225 with 8096 core locations) accepts unbraced frame with 1 to 8 bays and 1 to 48 stories.

II. SALES ENGINEERING DESIGN COMPARISONS

A. Allowable Stress versus Plastic Design for 24 Story Unbraced Frame C (Ref. 9, Chapter 19)

Material weight and cost data are compared in Table 1 for 24 stories complete and for Level 20 only. Conclusions from this table are as follows:

1. Most of material cost (73 percent) is in columns. Columns in bottom 14 stories account for over 60 percent of material cost. This makes weak-column-strong-beam approach economically attractive.
2. Material cost per square foot at Level 20 is 60 percent larger than same figure for entire frame. (Divide unit figures for Level 20 by 1.6 to extrapolate to 24 story frame)
3. The ASD/PD ratios in columns (3) and (6) indicate that <u>total</u> material weight and cost figures have nearly the same ratio for Level 20 as for entire frame. Thus, comparison of different designs for Level 20 gives results which are reasonably valid for the entire frame.
4. The ASD/PD ratios in columns (3) and (6) also indicate that plastic design seems to do a somewhat better job of reducing girder material weight and cost in the lower stories than in the upper stories. For example, plastic design saved 17 percent over allowable stress design for the A36 girders in the entire frame, and 30 percent at Level 20. (Compare columns (3) and (6) in row (1)).

B. Cost comparisons for Level 20 of unbraced Frame C.

Three different plastic designs for the girders on Level 20 and the columns below are included in Table 2. The designs are identified as follows:

Design 1 from Ref. 9, sway subassemblage method

Design 2 from Ref. 12, sway subassemblage method

Design 3 from Ref. 10, minimum plastic moment weight function method

Data from the allowable stress design in Ref. 9 is included at the top right in Table 2 for reference. The same unit material cost figures are used in Table 1 and 2.

1. Material cost summary from Table 2

Design No.	1	2	3	4
Material Cost	$1237	$1012	$1169	$1350
Cost Ratio (Col. N/Col. 2)	1.22	1.00	1.15	1.33
Column Steel	A441	A36	A441	A441

2. Cost ratio for Design 1 indicates the progress made in less than one year of experience with sway subassemblage method.

3. Cost ratio for Design 3 shows that minimum plastic moment weight function method does not necessarily lead to minimum cost. However this method does tend to indicate a more economical relative distribution of moment capacity. Design 3 made a cost saving on the A36 girders over Design 2 but lost this saving on the A441 columns.

4. To assist in evaluation of the method in Ref. 10 we need
   a. Design for Level 20 using A36 columns
   b. Sway subassemblage analyses at Level 20 for both A36 and A441 column designs. These analyses should indicate whether a minimum weight function design does or does not satisfy frame stability requirements. The allowance for $P\Delta$ effects used in Ref. 10 could also be evaluated from the sway analyses.
5. The constant girder depth used in Designs 1 and 2 promotes cost savings in fabrication and construction depth (and the cost of all vertical services and architectural material). These savings may be even more attractive than material cost savings.
6. The A36 columns in Design 2 promote welding cost savings over the A441 columns in the other designs.
7. Plastic Design 2 gives a 33 percent saving in material cost and a 6 inch reduction in construction depth over the comparable allowable stress design at Level 20. This is a nutshell summary of the dollar value of plastic design for multi-story frames.

III. STRUCTURAL ENGINEERING DESIGN COMPARISONS

A. Data on frame behavior for unbraced Frame C at Level 20 is summarized in Table 2, for Designs 1 and 2. This data includes:

1. Order of hinge formation.
    a. Solid circle indicates hinge which forms before or at ultimate load.
    b. Open circle indicates hinge which forms between ultimate load and sway mechanism load.
2. Axial load (P/Py) and moment (M/M_{pc}) ratios for column C at ultimate load.
3. Shear versus sway data at
    a. Working load
    b. 1.3 (Working load)
    c. Ultimate load
    d. Mechanism load

B. Hinge formation

1. The first hinges to form are at the leeward end of the three girders.
2. Each frame reaches its ultimate wind shear capacity when the fourth hinge forms at the windward end of the center (most stiff) girder. Thus, only 4 of the minimum of 6 hinges required for a sway mechanism actually form at ultimate load. The hinge pattern at ultimate load is definitely restricted, relative to any possible hinge pattern for a sway mechanism.

C. Load and moment ratios.

Column C is the most heavily loaded column in the story below Level 20. The P/Py and M/M_{pc} ratios for this column at ultimate load indicate the extent to which its capacity is utilized.

1. Column C in Design 1 is definitely in the strong-column-weak-beam category. All of the A441 columns below Level 20 could probably be reduced in size.
2. Column C in Design 2 is used more efficiently. This column is close to the weak-column-strong-beam category although the adjacent leeward girder hinge still tends to limit the moment in column C.

D. Shear versus sway data.

1. The working load sway for Design 1 is only 57 percent of that for Design 2. This increase in sway stiffness was obtained by strengthening the columns at a material cost increase of $225 or 22 percent. Sway stiffness costs dearly if it is provided by strengthening columns.
2. The ultimate load shear capacity of Design 1 is sufficient to carry a 27 psf working load wind with a load factor of 1.3. The corresponding figure for Design 2 is 22 psf. These wind capacities are 35 and 10 percent larger than the 20 psf design wind. On the other hand, the shear capacity when a mechanism forms is nearly the same for both designs and is just adequate to carry the 20 psf working load wind. This indicates that the shear capacity when a mechanism forms may not give a true estimate of the ultimate shear capacity. It also suggests that a design method which is based on ultimate load conditions rather than the mechanism load

condition may be both more rational and more economical. The plastic moment balancing method may be applied to either ultimate or mechanism load conditions with little essential modification by specifying restricted hinge patterns.

3. The sway deflection at the ultimate load condition appears to be more consistent than the sway deflection at the mechanism condition. This is indicated in the following sway deflection summary:

Design No.	1	2
Δ/h at ultimate load	0.006	0.005
Δ/h at mechanism load	0.020	0.009

If $P\Delta$ effects are to be included in a preliminary design, it is helpful to have reasonably consistent sway deflection data. This again suggests that the ultimate load condition may be a more reliable design criterion than the mechanism condition. The results of more sway subassemblage analyses like that described in Ref. 12 are needed to verify this tentative conclusion.

E. Story shear versus sway behavior for Level 20 of unbraced Frame C.

Shear versus sway curves for Designs 1 and 2 are shown in Fig. 1 which also indicates data at

1. Working load
2. 1.3 (Working load)
3. Ultimate load

4. Mechanism load
5. Formation of each plastic hinge

F. The ascending (stable) branch of the shear-sway curves are shown with a heavy line to distinguish them from the descending (unstable) branch. The same inelastic column theory and elastic-plastic girder theory is used for both branches. However, the descending portions of the shear-sway curves for a real frame may be influenced to some degree by several factors:

1. Strain hardening at first formed (leeward girder) hinges.
2. A minimal amount of ductile cladding.
3. Differential settlement of columns.
4. Initial crookedness of columns.
5. Axial load shortening of columns.
6. Shear distortion of joints.
7. Local buckling.
8. Axial loads in girders.

These factors would tend to be more active after ultimate load than before this load. The first two factors would raise and the last six factors would lower the unstable branch of the shear-sway curves. The net result is that the shear versus sway behavior beyond ultimate load for a real building frame must be considered as indefinite and unessential for many practical purposes. This again suggests that the ultimate load condition is a more rational and reliable design criterion than the mechanism load condition.

G. The $P\Delta$ effect is sometimes considered in design calculations by adding 2 percent of the gravity load in a story to the story shear (Ref. 10). This 2 percent rule is graphically illustrated at the lower right corner of the graph in Fig. 1. The rule is based on an assumed sway deflection index $\Delta/h = 0.020$. This 2 percent rule appears to have little relationship to the behavior of Designs 1 and 2.

H. The stronger columns in Design 1, relative to Design 2 result in the following:

1. A stiffer, stronger frame.
2. Earlier formation of plastic hinges at the leeward end of girders.
3. A more gradual approach to the ultimate load condition (more rounded knee in the shear-sway curve) and therefore more warning of collapse.
4. A considerable increase in ductility as measured by the mechanism sway deflection.

If these effects are considered desirable, they may be obtained at a cost of $225 at Level 20.

I. In view of the excess shear capacity in Design 1 it is of some interest to consider in an approximate manner the ability of this design to resist an earthquake shear. The seismic requirements of Ref. 13 are summarized below Fig. 1. If we divide the static ultimate shear capacity below Level 20 (204 kips) by the factored seismic shear ($1.3\ H_{20} = 354$ kips) the result suggests that Design 1 could carry 58 percent of the equivalent static seismic shear required by the SEAOC Code (Ref. 13). It should be emphasized that the seismic factors in Ref. 13 are not intended for use in a plastic design approach. The dynamic elastic-plastic response of the clad and dampened building above and below Level 20 may be expected to influence seismic effects at this level. More study is needed to justify the use of elastic static seismic factors in a plastic design. There is some reason to expect that the increased ability of a ductile steel frame to absorb energy in the elastic-plastic range may justify smaller static seismic factors than those deduced from elastic behavior. If this is true then Design 1 may not be far removed from an adequate aseismic design.

J. The shear-sway curves in Fig. 1 may be used to compare the capacity for energy absorption of Designs 1 and 2. The energy calculations are carried out in Table 3 where it is assumed that energy capacity is the area under the shear versus Δ/h curve times the story height. The energy capacities (ECap) are:

Design No.	1	2
ECap at ultimate load (ft.-kips)	10.8	5.6
ECap ratio (Col. N/Col. 2)	1.93	1.0
ECap at mechanism load (ft.-kips)	40.6	13.0
ECap ratio (Col. N/Col. 2)	3.12	1.0

The ECap ratio at ultimate load for Design 1 reflects the energy capacity increase provided by the stronger columns in this design. An even larger ECap ratio at the mechanism load results from the condiderably increased ductility of Design 1 relative to Design 2. We can increase the energy capacity at Level 20 by a factor of 3 for $225 in column material. The cost of ductility in steel is an attractive bargin in aseismic design.

IV. CONCLUSIONS

The many and diverse requirements which may challenge the designer of an unbraced multi-story frame suggest that he needs a flexible and adaptable design method to meet the challenge. Economy and rationality recommend the plastic design approach. The flexibility and adaptability of the plastic moment balancing method help to make the plastic design approach a more potent and practical procedure.

TABLE 1

MATERIAL WEIGHT AND MATERIAL COST COMPARISONS

FOR UNBRACED FRAME C

(Ref. 9, Chapter 19)

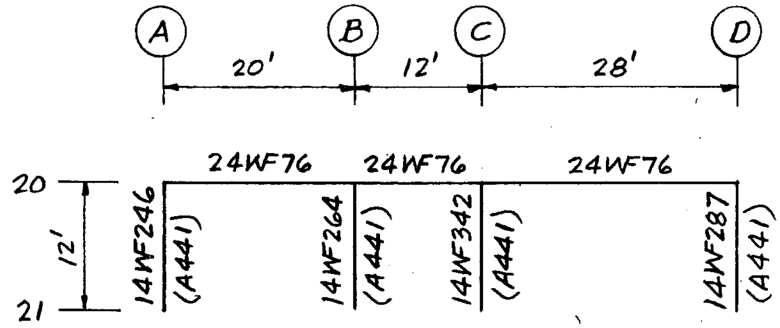
Row	Design Method Item	24 Stories Complete: Allow Stress (1)	Plastic (2)	Ratio ASD/PD (3)	Level 20 Only: Allow Stress (4)	Plastic (5)	Ratio ASD/PD (6)
	Material Weight (tons)						
(1)	A36 girders	50.2	42.8	1.172	2.97	2.28	1.301
(2)	A36 columns	24.5	23.0	1.065	--	--	--
(3)	A441 columns	85.0	83.4	1.020	7.08	6.84	1.036
(4)	Totals	159.7	149.2	1.069	10.05	9.12	1.101
(5)	Totals (lb./sf)	9.21	8.62		13.96	13.35	
	Material Cost ($)						
(6)	A36 girders	5,570	4,750	1.172	330	253	1.301
(7)	A36 columns	2,720	2,550	1.065	--	--	--
(8)	A441 columns	12,230	12,000	1.020	1020	984	1.038
(9)	Totals	20,520	19,300	1.062	1350	1237	1.092
(10)	Totals ($/sf)	0.59	0.56		0.94	0.86	

Material cost data: (Ref. 9, Chapter 12)

ASTM A36 W and H shapes $111/ton

ASTM A441 H shapes $144/ton

TABLE 2
DESIGN COMPARISONS – FRAME C AT LEVEL 20



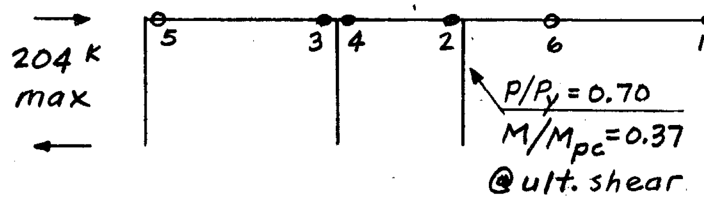
DESIGN 1 (Ref. 9, Chapter 19)

AS Design – (Ref. 9, Chapter 19)		
Girders (A36)	2.97 T @ \$111 =	\$ 330
Columns (A441)	7.08 T @ \$144 =	1020
Material	10.05 T	\$ 1350

Girders (A36) 2.28 T @ \$111 = \$ 253

Columns (A441) 6.84 T @ \$144 = 984

Material	9.12 T Weight	\$ 1237 Cost

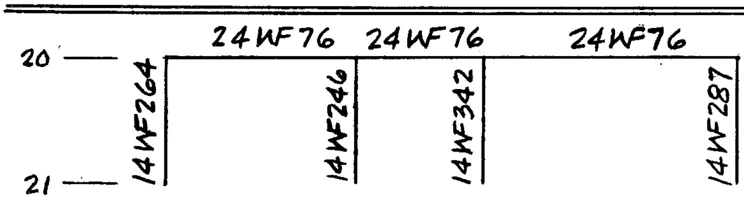


Wind shear = 114 k $\frac{\Delta}{h} = 0.0015$

1.3 (Wind shear) = 148 k $\frac{\Delta}{h} = 0.0025$

Ultimate shear = 204 k $\frac{\Delta}{h} = 0.006$

Mechanism shear = 151 k $\frac{\Delta}{h} = 0.020$

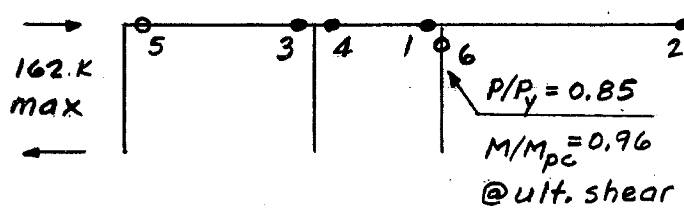


DESIGN 2 (Ref. 12)

Girders (A36) 2.28 T @ \$111 = \$ 253

Columns (A36) 6.84 T @ \$111 = 759

Material	9.12 T Weight	\$ 1012 Cost

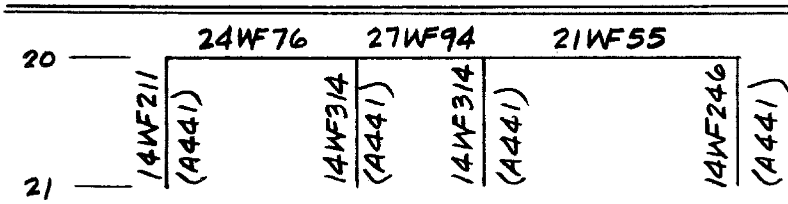


Wind shear = 114 k $\frac{\Delta}{h} = 0.0027$

1.3 (Wind shear) = 148 k $\frac{\Delta}{h} = 0.0038$

Ultimate shear = 162 k $\frac{\Delta}{h} = 0.005$

Mechanism shear = 146 k $\frac{\Delta}{h} = 0.009$

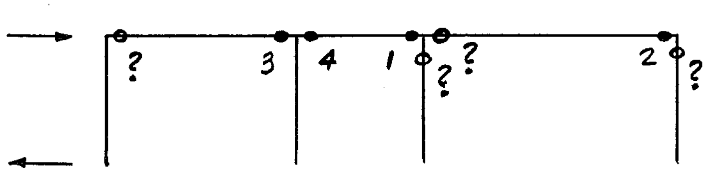


DESIGN 3 (Ref. 10)

Girders (A36) 2.09 T @ \$111 = \$ 232

Columns (A441) 6.51 T @ \$144 = 937

Material	8.60 T Weight	\$ 1169 Cost



Hinge order estimated

Shear vs. sway data not available

TABLE 3

CAPACITY FOR ENERGY ABSORPTION

Assume ECap = (area under shear versus Δ/h) x h

Design 1 (A441 columns, A36 girders, Material cost $1237)

(1) Shear (kips)	(2) Δ/h (x1000)	(3) δ(Δ/h)	(4) Avg. Shear (kips)	(5) δ(ECap)/h	(6) (ECap)/h (x1000)	Remarks
		1.0	47.5	42.5		
95	1.0				47.5	
		1.0	116.5	116.5		
138	2.0				164.0	
		1.0	197.5	197.5		
157	3.0				361.5	
		3.0	180.5	541.5		
204	6.0				903.0	Ultimate
		14.0	177.5	2485.0		
151	20.0				3388.0	Mechanism

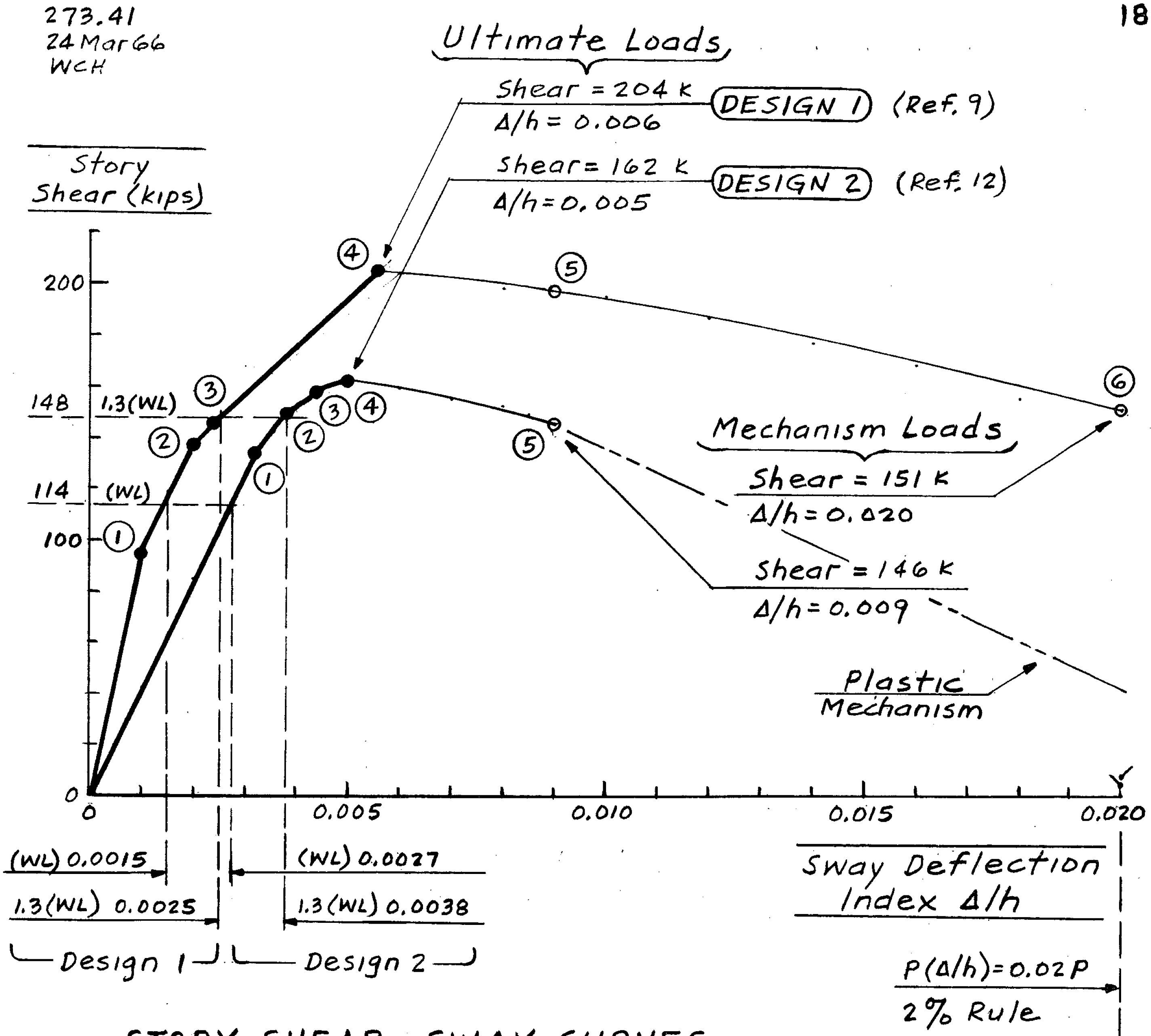
At ultimate load	ECap = 10.8 ft.-kips	(26.5%)
During mech. sway	ECap = 29.8	(73.5%)
At mechanism load	ECap = 40.6 ft.-kips	

Design 2 (A36 columns and girders, Material cost $1012)

(1) Shear (kips)	(2) Δ/h (x1000)	(3) δ(Δ/h)	(4) Avg. Shear (kips)	(5) δ(ECap)/h	(6) (ECap)/h (x1000)	Remarks
		3.2	61.0	195.2		
132	3.2				195.2	
		0.6	140.0	84.0		
148	3.8				279.2	
		0.6	153.0	91.8		
158	4.4				371.0	
		0.6	160.0	95.7		
162	5.0				467.0	Ultimate
		4.0	154.0	616.0		
146	9.0				1083.0	Mechanism

At ultimate load	ECap = 5.6 ft.-kips	(43%)
During mech. sway	ECap = 7.4	(57%)
At mechanism load	ECap = 13.0 ft.-kips	

Conclusion: Design 1 increases capacity for energy absorption by factors of 1.93 at ultimate load and 3.12 at mechanism load, relative to design 2, at a material cost increase of $225 (22 percent).



STORY SHEAR - SWAY CURVES
FRAME C - LEVEL 20
for Designs 1 and 2

FIGURE 1

Siesmic Forces - Frame C SEAOC Code (Ref. 13)

Fundamental period	$T = 0.1N = 0.24$ sec
Base shear coefficient	$C = 0.05/\sqrt[3]{T} = 0.0806$
Horizontal force factor	$K = 0.67$
Total dead load	$W = 5200$ kips (24 stories)
Base shear	$V = KCW = 281$ kips
Story shear below level 20	$H_{20} = 272$ kips

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